GRAPHING NONLINEAR FUNCTIONS

General Transformations on graphs

If given a sketch of the graph of $f(x)$ …

- $-f(x)$ is a vertical reflection (across x-axis)
- $f(-x)$ is a horizontal reflection (across y-axis)
- $f(x)+c$ is a vertical shift, up c units ($c > 0$)
- $f(x)-c$ is a vertical shift, down c units ($c > 0$)
- $f(x+c)$ is a horizontal shift, left c units ($c > 0$)
- $f(x-c)$ is a horizontal shift, right c units ($c > 0$)

(You only need to know the ones in the box – the rest are FYI).

- $a \cdot f(x)$ is a vertical stretch if $a > 1$
- $a \cdot f(x)$ is a vertical shrink if $0 < a < 1$
- $f(ax)$ is a horizontal shrink if $a > 1$
- $f(ax)$ is a horizontal stretch if $0 < a < 1$

Graphs of Exponential and Logarithmic Functions

- The graph of $f(x) = a^x$ ($a > 0$, $a \neq 1$) always passes through the point (0, 1), and has this general shape:

- The graph of $f(x) = \log_a x$ ($a > 0$, $a \neq 1$) always passes through the point (1, 0), and has this general shape:

- The functions a^x and $\log_a x$ are *inverse functions* of each other. Therefore, their graphs are reflections of each other across the line $y = x$ (true of all inverse functions).

- Use the transformations described above (reflections and shifting) to sketch functions such as $f(x) = 3^{x-5} + 2$ and $g(x) = -\log_4 x$.

NOTE: Notice on the graph of $f(x) = \log_a x$ that the domain is $\{x \mid x > 0\}$ (i.e., only positive x's are used). It is evident from this graph that **you cannot take the log of a negative number** (or zero).

Therefore, when solving equations such as $\log_3(x-2) + \log_3(x+6) = 2$, we eliminate any answers that would cause us to take the log of a negative number or zero.

Graphing Quadratic Functions (Parabolas – Sections 8.6 and 8.7)

1) Quadratic Function of the form $f(x) = a(x-h)^2 + k$:

Vertex is located at the point (*h*, *k*).

2) Quadratic Function of the form $f(x) = ax^2 + bx + c$:

Vertex (*h*, *k*) is located at $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

- In BOTH cases: $a > 0 \Rightarrow$ parabola opens *upward*, so the vertex represents the minimum *y*-value.
  $a < 0 \Rightarrow$ parabola opens *downward*, so the vertex represents the maximum *y*-value.

Form (1) can be converted to Form (2) by FOILing and simplifying. Form (2) can be converted to Form (1) by "completing the square".

Graphing Conic Sections (Chapter 10)

NOTE: We are skipping graphs of $x = a(y-k)^2 + h$ (sideways parabolas)

Circle:

$(x-h)^2 + (y-k)^2 = r^2$, where the center is (*h*, *k*), and *r* = radius

Ellipse: (centered at origin)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ [Plot the points $(\pm a, 0)$ and $(0, \pm b)$ to sketch the ellipse; make sure the equation is equal to 1 first!]

Hyperbola: (centered at origin)

1) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ [Plot the points $(\pm a, 0)$ and $(0, \pm b)$ to sketch the rectangle and its diagonals; vertical branches touch either side of rectangle]

2) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ [Plot the points $(\pm a, 0)$ and $(0, \pm b)$ to sketch the rectangle and its diagonals; horizontal branches touch top and bottom of rectangle]

Remember it like this: If the *x* comes first, the hyperbola has *x*-intercepts. If the *y* comes first, the hyperbola has *y*-intercepts.