Imperial Valley College
Math Lab

Functions: Composition and Inverse Functions

FUNCTION COMPOSITION

In order to perform a composition of functions, it is essential to be familiar with function notation.
If you see something of the form "$f(x) =$ [expression in terms of x]", this means that **whatever you see in the parentheses following f should be substituted for x in the expression**. This can include numbers, variables, other expressions, and even other functions.

EXAMPLE: $f(x) = 4x^2 - 13x$

$f(2) = 4 \cdot 2^2 - 13(2)$

$f(-9) = 4(-9)^2 - 13(-9)$

$f(a) = 4a^2 - 13a$

$f(c^3) = 4(c^3)^2 - 13c^3$

$f(h+5) = 4(h+5)^2 - 13(h+5)$

Etc.

A **composition of functions** occurs when one function is "plugged into" another function.

The notation $(f \circ g)(x)$ is pronounced "f of g of x", and it literally means $f(g(x))$.
In other words, you "plug" the $g(x)$ function *into* the $f(x)$ function.

Similarly, $(g \circ f)(x)$ is pronounced "g of f of x", and it literally means $g(f(x))$.
In other words, you "plug" the $f(x)$ function *into* the $g(x)$ function.

WARNING: Be careful not to confuse $(f \circ g)(x)$ with $(f \cdot g)(x)$, which means $f(x) \cdot g(x)$.

EXAMPLES: $f(x) = 4x^2 - 13x$ $\quad$ $g(x) = 2x + 1$

a. $(f \circ g)(x) = f(g(x)) = 4[g(x)]^2 - 13 \cdot g(x) = 4(2x+1)^2 - 13(2x+1)$
$= [simplify] \ldots = 16x^2 - 10x - 9$

b. $(g \circ f)(x) = g(f(x)) = 2 \cdot f(x) + 1 = 2(4x^2 - 13x) + 1 = 8x^2 - 26x + 1$

A function can even be "composed" with itself:

c. $(g \circ g)(x) = g(g(x)) = 2 \cdot g(x) + 1 = 2(2x+1) + 1 = 4x + 3$

INVERSE FUNCTIONS

The notation for inverse functions can cause confusion. **It is important to know that** $\boldsymbol{f^{-1}(x) \neq \frac{1}{f(x)}}$ **.**

Instead, $f^{-1}(x)$ indicates the **inverse function of** $\boldsymbol{f(x)}$, which can be thought of as the function that "reverses" $f(x)$, or "undoes" everything that $f(x)$ does.

EXAMPLE: Let $f(x) = 3x - 1$.

In words, $f(x)$ takes a number, *multiplies it by 3*, and then *subtracts 1* from it.

The ***opposite*** or ***reverse*** of this procedure would be to *add 1* to a number, then *divide it by 3*.

For instance, $f(\mathbf{4}) = 3 \cdot 4 - 1 = \mathbf{11}$. **Input = 4 , Output = 11**

In reverse, take the output 11, add 1 to it, then divide by 3: **11** + 1 = 12

$12 \div 3 = \mathbf{4} =$ input .

In other words, the output became the input, and vice-versa. They switched roles!

Mathematically, In each ordered pair (x, y) associated with a function, the *x* and *y* "switch places": $(x, y) \rightarrow (y, x)$

In the above example, $(4, 11)$ became $(11, 4)$.

Procedure:

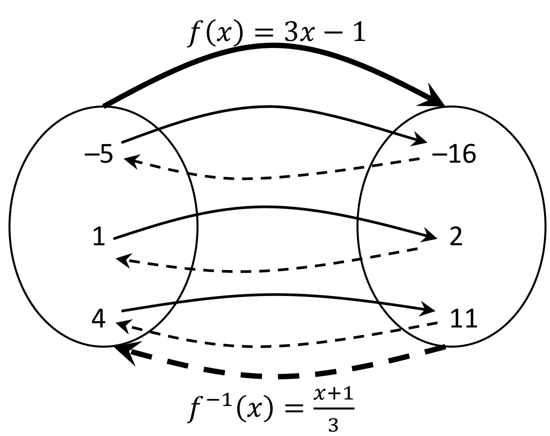
Given $f(x)$, take the following steps to find $f^{-1}(x)$:

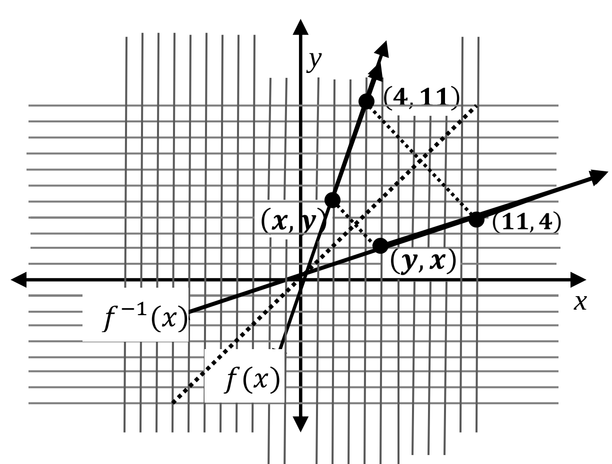
1) Replace $f(x)$ with *y* in the equation
2) Swap the *x* and *y* in the equation (they "trade places")
3) Solve the equation for *y* (i.e., isolate the *y*)
4) Once *y* is isolated, replace it with $f^{-1}(x)$.

EXAMPLE:

1) $y = 3x - 1$
2) $x = 3y - 1$
3) $x + 1 = 3y \rightarrow \frac{x+1}{3} = y$
4) $f^{-1}(x) = \frac{x+1}{3}$ or $f^{-1}(x) = \frac{1}{3}x + \frac{1}{3}$

Check that these values work in both directions:





Notice how **the graphs of** $\boldsymbol{f(x)}$ **and** $\boldsymbol{f^{-1}(x)}$ **are symmetric (mirror images) across the diagonal line *y* = *x***, which is the result of all the *x*- and *y*-values "trading places". Imagine each (x, y) point as "hopping" across the line *y* = *x*, becoming (y, x).