Setting the Signs Properly when Factoring *2nd* Degree Trinomials

Conditions: Variables ***x*** & ***y***, and coefficients a, b, & c $\in \mathbb{R}$ such that $a > 0$ & $c \neq 0$.

Case *I*: $ax^2 + bxy + cy^2$ *Expanded Form*

$= (\quad + \quad)(\quad + \quad).$ Factored Form

Case *II*: $ax^2 - bxy + cy^2$ *Expanded Form*

$= (\quad - \quad)(\quad - \quad).$ Factored Form

*Common Error Set-up in **Case II***: $ax^2 - bxy + cy^2 \neq (\quad - \quad)(\quad + \quad).$

Case *III*: $ax^2 + bxy - cy^2$ *Expanded Form*

$= (\quad + \quad)(\quad - \quad),$

OR Factored Form

$= (\quad - \quad)(\quad + \quad).$

Case *IV*: $ax^2 - bxy - cy^2$ *Expanded Form*

$= (\quad - \quad)(\quad + \quad),$

OR Factored Form

$= (\quad + \quad)(\quad - \quad).$

*Common Error Set-up in **Case IV***: $ax^2 - bxy - cy^2 \neq (\quad - \quad)(\quad - \quad).$

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Conditions: Variables ***x* & *y***, and coefficients a, b, & c $\in \mathbb{R}$ such that $a > 0$ & $c \neq 0$.

Ex. 1:

$x^2 + 8x + 15,$ for a = +1, b = +8 & c = +15.

Then, the factorization for this trinomial follows **Case *I***:

$= (x + 3)(x + 5).$ Therefore, the sign set-up is positive in each pair of parentheses.

Ex. 2:

$3x^2 - 7xy + 4y^2,$ for a = +3, b = −7 & c = +4.

Now, the trinomial factorization is by **Case *II***:

$= (3x - 4y)(x - y).$ Therefore, the sign set-up is negative in each pair of parenthesis.

Ex. 3:

$2x^2 + 4x - 6$ for a = +2, b = +4 & c = −6.

This trinomial factorization is similar to **Case *III***:

$= (x + 3)(2x - 2).$ Therefore, the sign set-up alternates with a positive in one pair of parentheses and a negative in the other pair of parentheses.

Ex. 4:

$\frac{1}{4}x^2 - \frac{1}{24}xy - \frac{1}{2}y^2$ for $a = +\frac{1}{4}$, $b = -\frac{1}{24}$ & $c = -\frac{1}{2}$.

The trinomial factorization matches with **Case *IV***:

$= \left(\frac{1}{2}x - \frac{3}{4}y\right)\left(\frac{1}{2}x + \frac{2}{3}y\right).$ Then, the sign set-up alternates with a positive in one pair of parentheses and a negative in the other pair of parentheses.

Ex. 5: (***Special Case***)

$-5x^2 + 11xy - 2y^2$ for a = −5, b = +11 & c = −2. Since a is negative, the first step is factoring a −1 by the ***GCF*** to switch that a to a positive coefficient.

$= -1(5x^2 - 11xy + 2y^2)$ Now, inside the parentheses, the coefficients are: a = +5, b = −11 & c = +2, and this trinomial ties with ***Case II***:

$= -1(5x - y)(x - 2y).$ Therefore, the sign set-up is negative in each pair of parentheses, and the factorization now has three factors:

−1, (5x − y), and (x − 2y).