FACTORING SUMMARY SHEET

Factoring techniques

- Factoring out the **GCF (Greatest Common Factor)**

Ex: $15x^3 - 20x^2 = 5x^2\left(\frac{15x^3}{5x^2} - \frac{20x^2}{5x^2}\right) = 5x^2(3x-4)$

Divide each term by the GCF

- Factoring by **grouping**

Ex: $4pq + 12p + q^2 + 3q = (4pq + 12p) + (q^2 + 3q) = 4p(q+3) + q(q+3)$

Note: You may need to arrange the terms into a different order first $= (q+3)(4p+q)$.

- **Trinomial** with leading coefficient = 1: $x^2 + bx + c$
  1) Find p, q such that $p + q = b$ and $pq = c$.
  2) The trinomial factors as $(x + p)(x + q)$

Ex: $x^2 - 6x - 16$ → Find p and q such that $p + q = -6$ and $pq = -16$.
In this case, $p = 2$ and $q = -8$ works (check it).
So, the trinomial factors as $(x+2)(x-8)$.

- **Grouping (key number) method** for **trinomial** with leading coefficient ≠ 1: $ax^2 + bx + c$

  1) Key number = ac
  2) Find p, q such that $p + q = b$ and $pq = ac$
  3) Re-write trinomial by 'splitting up' middle term: $ax^2 + \overbrace{px + qx}^{bx} + c$
  4) Factor by grouping: $(ax^2 + px) + (qx + c)$

Ex: $6x^2 + 13x + 5$

1) Key number = $6 \cdot 5 = 30$
2) In this case, $p = 3$ and $q = 10$ works (check that $p + q = 13$ and $pq = 30$).
3) Break up the middle term "$13x$" to $3x + 10x$ (or, $10x + 3x$ will work too):

$$6x^2 + \underbrace{3x + 10x}_{13x} + 5$$

4) Now factor by grouping:

$$(6x^2 + 3x) + (10x + 5)$$

$$= 3x(2x+1) + 5(2x+1)$$

$$= (2x+1)(3x+5) .$$

Of course, FOIL and check that $(2x+1)(3x+5) = 6x^2 + 13x + 5$. ✓
Always check your factorizations by multiplying them back together!

Special Factorizations

- **Perfect square trinomial**: $A^2 + 2AB + B^2 = (A+B)^2$

  Ex: $9x^2 + 24x + 16 = (3x)^2 + 2(3x)(4) + 4^2 = (3x+4)^2$ [$A = 3x$, $B = 4$]

- **Difference of Perfect Squares**: $A^2 - B^2 = (A+B)(A-B)$

  Ex: $25x^2 - 81 = (5x)^2 - 9^2 = (5x+9)(5x-9)$ [$A = 5x$, $B = 9$]

- **Sum/Difference of Perfect Cubes**:

$$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$$
$$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$$

  Ex: $27x^3 + 125 = (3x)^3 + 5^3 = (3x+5)((3x)^2 - (3x)(5) + 5^2)$

  $= (3x+5)(9x^2 - 15x + 25)$ [$A = 3x$, $B = 5$]

Combining Factoring Techniques

When in doubt for how to begin factoring…

- Check if you can factor out a GCF first.
- If the resulting polynomial has 4 terms, check if you can factor by grouping.
- If the resulting polynomial has 3 terms, check if it is a factorable trinomial.
- If the resulting polynomial has 2 terms, check if it is a difference of perfect squares or sum/difference of cubes.

Ex: $12m^3 - 147m = 3m(4m^2 - 49) = 3m((2m)^2 - 7^2) = 3m(2m+7)(2m-7)$

Factor out GCF = 3m *Now you have a difference of perfect squares*

Zero Factor Principle or Principle of Zero Products

If $AB = 0$, then $A = 0$ or $B = 0$ (A and B can be polynomials or other expressions).

When solving a polynomial equation of degree 2 or higher (i.e., not a linear equation)…

- Re-arrange the equation so that one side is equal to zero
- Try to factor the polynomial
- If the polynomial is factored, use the Principle of Zero Products.

Ex: $x^2 + 3x = 28 \rightarrow x^2 + 3x - 28 = 0 \rightarrow (x+7)(x-4) = 0$

$\rightarrow x + 7 = 0$ or $x - 4 = 0 \rightarrow x = -7$ or 4

****One of the most common algebra mistakes students make: “ $(A+B)^2 = A^2 + B^2$ ”**

It’s **not** true!!! $(A+B)^2 \neq A^2 + B^2$!!! Check: Does $(1+3)^2 = 1^2 + 3^2$? No.