Constructions of Isosceles and Equilateral Triangles

Part 1:

- What are the characteristics of an isosceles triangle? What properties of geometric figures do you know that might be of help to you to construct an isosceles triangle and guarantee that it is isosceles *without* using any measurement tool

  Hint: Recall the characteristics of isosceles triangles and how they might be related to properties of a circle.

- On paper, use a compass to construct a circle. Use the circle and a straightedge to construct two congruent segments with a common endpoint. Without measuring, how can you guarantee these segments are congruent? From these segments, construct an isosceles triangle.

- Now in a similar way, construct an isosceles triangle with Sketchpad.

To construct a circle:

- Construct a segment to represent the radius of the circle.
- Select one of the segment's endpoints, as the center, and the segment itself.
- Choose **Circle By Center + Radius** under the **Construct** menu.

To construct a point on the circle:

- Select the circle.
- Choose from the **Construct** menu, the **Point on Circle** command.

- Verify that the triangle you constructed is isosceles by checking the lengths of the triangle's sides. Measure the triangle's three interior angles, and classify your isosceles triangle.

- Investigate the sum of the measures of the three interior angles. Drag any vertex to manipulate your triangle. Observe what happens to the posted lengths and angle measurements. What conclusions can you draw from your observations? Discuss and prove a conjecture about angle measurement that describes your findings.

- Compare and contrast the effects of dragging *each* of the triangle's vertices around

your sketch window. How are the ways you may manipulate your triangle related to your choice of vertex?

- Discuss the differences between *drawing* and *constructing* a triangle.

- Save your sketch onto a disk as **isostri.gsp**. Keep the sketch open on your computer! (**Instructor Note**: A *Sketchpad* file illustrating the construction of an isosceles triangle has been saved as isostri.gsp.)

Part 2:

- What are the characteristics of an equilateral triangle? Drag one of the vertices of your *constructed* isosceles triangle until the triangle appears to be equilateral.

- Without using measurement tools, how can you confirm that this third side is congruent to the two radii of the circle?

- *Construct* an equilateral triangle that is guaranteed to always be equilateral without the use of any measurement tools. **Hint**: The **Point of Intersection** command under the **Construct** menu.

- Click and drag one of the vertices of your triangle. How does it affect the equilateral triangle?

- Verify that the triangle is equilateral by measuring the length of the sides and the interior angles of the triangle. Now drag one of the vertices. Observe what happens to the posted lengths and angle measurements. Write, discuss and prove a conjecture describing your findings.

Part 3:

Custom Tools are generalized recordings of sketches. It is sometimes convenient to record the steps of particular sketches (e.g., regular polygons) that you have made for subsequent playback. For example, when a future sketch requires a constructed equilateral triangle, you can just use your custom tool rather than reconstructing one. Sketchpad can generate a custom tool of a construction while you are sketching it. You can use custom tools repeatedly to generate figures, or portions of figures, while sketching. Individual custom tools can be used as foundations for larger sketches, leading to potentially more complex constructions.

- Create a custom tool for the construction of an equilateral triangle using the construction procedure you used in Part 2. (**Instructor Note**: A *Sketchpad* file illustrating the construction of an equilateral triangle has been saved as equil.gsp.)

Creating a Custom Tool:

- Create a sketch of an example of the geometric construction you want the tool to produce. You may use any Sketchpad tools or menus to create this exemplar.
- Select both the given objects (usually, independent points) and the desired resulting objects you'd like the tool to produce. The order in which you select the givens determines the order in which you'll match the givens when using the tool.
- Click and hold on the **Custom tools icon**  in the Toolbox. Choose **Create New Tool** from the menu that appears.
- You may name the tool in the dialog box that appears, and click OK.
- Your tool is added to the **Custom Tools** menu, and is ready to use.

Using your Equilateral Triangle Custom Tool:

- Click and hold on the **Custom tools icon**  in the Toolbox. The **Custom Tools** menu appears.
- Choose your equilateral triangle tool from the menu.
- Move your mouse over the sketch and click in two different places. An equilateral triangle appears in your sketch window.
- When you're finished making an equilateral triangle, click on any other tool in the **Toolbox** or press the **Esc** key.

- Use your equilateral triangle custom tool to create a pattern in your sketch window.

- What are the benefits of having **Custom Tools**? Think of an example where using a **Custom Tool** would be helpful to your students.

Extensions:

- Euclid's first proposition in the *Elements* is, "For a line segment AB, there is an equilateral triangle having the segment as one of its sides". He provides a proof using a similar construction to the one you have just completed. Provide your own proof that the triangle you constructed is an equilateral triangle.

- Create a custom tool for the construction of a right triangle. Make sure to drag each vertex to confirm it stays a right triangle. *Be sure to save it for future use on your disk.* Describe how you constructed your right triangle. You may find the **Perpendicular Line** command under the **Construct** menu useful. (**Instructor Note**: A *Sketchpad* sketch illustrating the construction of a right triangle has been saved as righttri.gsp.)