

# Geometer's Sketchpad

## Exploring Characteristics of Triangles



### Part 1:

- Discuss the characteristics of a triangle.
- Draw a triangle in your sketch window. There are several ways to draw a triangle using *The Geometer's Sketchpad*. One way is to use segments to connect three non-collinear points.

#### To create a new sketch window:

- Under the **File** menu, choose **New Sketch**.

#### To create three points:

- Choose the **Point Tool**  by clicking and letting go. You will see the Point Tool highlighted, meaning you've selected it as your active tool.
- Move the pointer into the sketch window and click to create a point.
- Move the pointer to where you want to create a second point and click. Do the same for a third. Note that the last point you created is still selected (i.e., it has a bold pink outline).
- Choose the **Selection Arrow Tool**  from the toolbox, and click anywhere in the sketch window. This should de-select the third point.

#### To construct a segment:

- Select two points. Position the tip of the **Selection Arrow** over one of the points, and click to select it. To select the second point, position the tip of the selection arrow over it and click.
- Under the **Construct** menu, select **Segment**. Repeat this procedure to construct the remaining two sides of the triangle. Remember, click anywhere in the sketch window to de-select all objects.

- Once a triangle is drawn we can manipulate it. Click on one of the triangle's vertices (or sides), hold down your mouse button and drag it. What happens to your triangle?

### Part 2:

Exercise 1

- Click on a vertex of your triangle and drag it to form a right triangle. How can you be sure that your triangle is a right triangle?
- Verify that your triangle is a right triangle by measuring its interior angles. Among other things, Sketchpad is capable of measuring angles and lengths of segments. Prior to measuring any objects, you might want to select a standard unit of measure for both distance and angle measurement.

**To select a standard unit of measure:**

- Under the **Edit** menu, select **Preferences**.
- Choose a unit for angle measurement and distance, and the displayed precision for both.
- Click the **OK** button.

**To measure an angle:**

- Select three points that define the angle. Here, selection order is important! You must select: point, vertex, point.
- Choose the **Angle** command under the **Measure** menu. The measure of the angle will appear in your sketch window.



- Drag your triangle around to make several different right triangles.
- Measure all three sides and interior angles of your triangle.

**To measure the length of a segment:**

- Select the segment.
- Choose from the **Measure** menu, the **Length** command.

- Draw an isosceles triangle. Confirm that you have an isosceles triangle by measuring the lengths of the sides of the triangle. If your triangle is not isosceles, drag one of its vertices until it becomes isosceles.
- As you drag the sides or vertices of your triangle, you might notice that your displayed measurements might be slightly off from what you expect. How can you explain this?
- Label your triangle as "isosceles" in a text box in your sketch window.

**To create a text box:**

- Select the **Text Tool**  in the tool box.
- Double-click anywhere inside your sketch window.
- When a flashing cursor appears, you may type in the text box.
- When you are finished typing, click anywhere outside of your text box.
- To edit your text box, select either the **Selection Arrow**  or the **Text Tool** and double-click inside the text box. A flashing cursor should appear, allowing you to edit.

Part 3:

- Draw a sketch and record the three angle measurements of a scalene, obtuse triangle in your sketch window. What is the sum of the three interior angles of your triangle? Compare the sum with your neighbor's sum. Comment on your findings.
- Use Sketchpad to calculate the sum of the three interior angle measures.

**To use the Sketchpad's built-in calculator:**

- With the **Selection Arrow**, select all three angle measures.
- Choose **Calculate** under the **Measure** menu.
- The angle measures have been stored under the **Values** pull down menu and can be moved into the calculator display by highlighting.
- Use those measures and the calculator keypad to form an expression for the sum of the three angle measures in the calculator display.
- To evaluate the expression, click on the **OK** box.

- What is the sum of the measures of the three angles? Drag any vertex to manipulate your triangle. What do you observe about the angle measurements and the sum while manipulating your triangle? What conclusions can you draw from your observations? (**Instructor Note:** A *Sketchpad* file illustrating the sum of the interior angles has been saved as [angles.gsp](#))
- Do these manipulations and observations constitute a valid proof? Explain your reasoning.
- Formally prove that the sum of the interior angles of a triangle is  $180^\circ$ .

Extensions:




- We have formally proven that the sum of the interior angles of a triangle is  $180^\circ$ . We would like to further investigate the sum of the interior angles of other polygons. In a new sketch,

Exercise 1

draw a quadrilateral and find the sum of its interior angles. What do you observe about the angle measurements and the sum while manipulating your quadrilateral? Is the sum of the interior angles of any quadrilateral equal? Explain your answer. What mathematical conclusions can you draw from your observations? Investigate a concave quadrilateral. What conclusions can you draw about The Geometer's Sketchpad from your observations?

- Continue the investigation using both pentagons and hexagons. On paper, organize the sum of the interior angles of each of the four figures in a table. Predict the sum of the interior angles of an octagon. Draw an octagon in a new sketch window and assess your prediction. Generalize the pattern to produce a formula for the sum of the interior angles of an n-gon. Verify your formula by an informal proof.
- In each of your drawn polygons, investigate the sum of the *exterior* angles. What do you observe about the sum while manipulating your polygon? Discuss and write a conjecture about your findings. Why is the sum of the exterior angles of a polygon not dependent on the number of sides of that polygon? Explain your reasoning by a formal or informal proof.

**To extend segments to rays:**

- Point your selection arrow on the **Point Tool**  in the Toolbox.
- Hold down your mouse button to view all three straightedge options  and select the **Ray**  option.
- **Select** the initial point of your ray and another point for the ray to go through.